Lecture 02b – Sampling designs II

ENVX2001 Applied Statistical Methods

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Outline

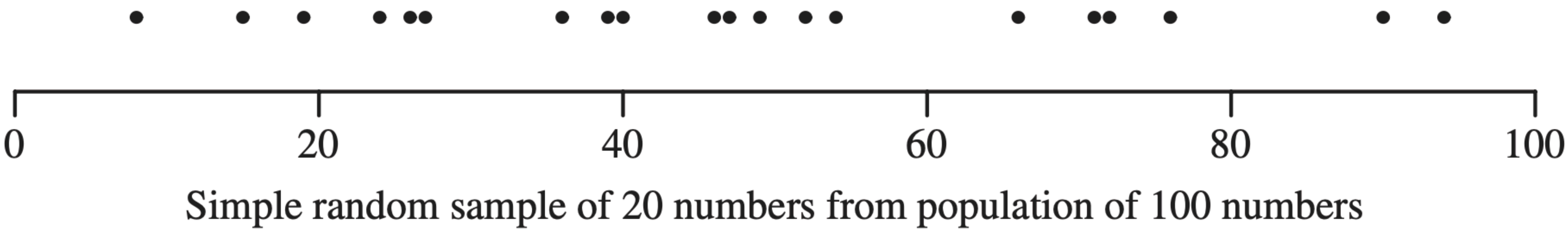
- Welcome back!
- In the last lecture…
- ~~Simple~~ Stratified random sampling
- Stratified random sampling: estimates
- Data story: soil carbon
- Comparison
- Monitoring

Welcome back!

In the last lecture…

- We learned about **simple random sampling**
- Each unit had an equal chance of being selected
- We calculated confidence intervals for population estimates
- We saw some limitations of this approach (not always representative)

Simple random sampling



Each unit has an equal chance of being selected.

Not *always* the case, but still a good technique.

Simple random sampling

Each unit has an equal chance of being selected.

Not *always* the case, but still a good technique.

Simple random sampling: potential problems

Imagine tossing 10 random points onto a landscape.

By pure chance...

- We might miss some important areas entirely
- Or sample some areas too much

This is more likely when:

- Sample size is small
- The landscape has distinct zones

Simple random sampling: theoretical example

If an area has:

- 80% grassland
- 20% wetland

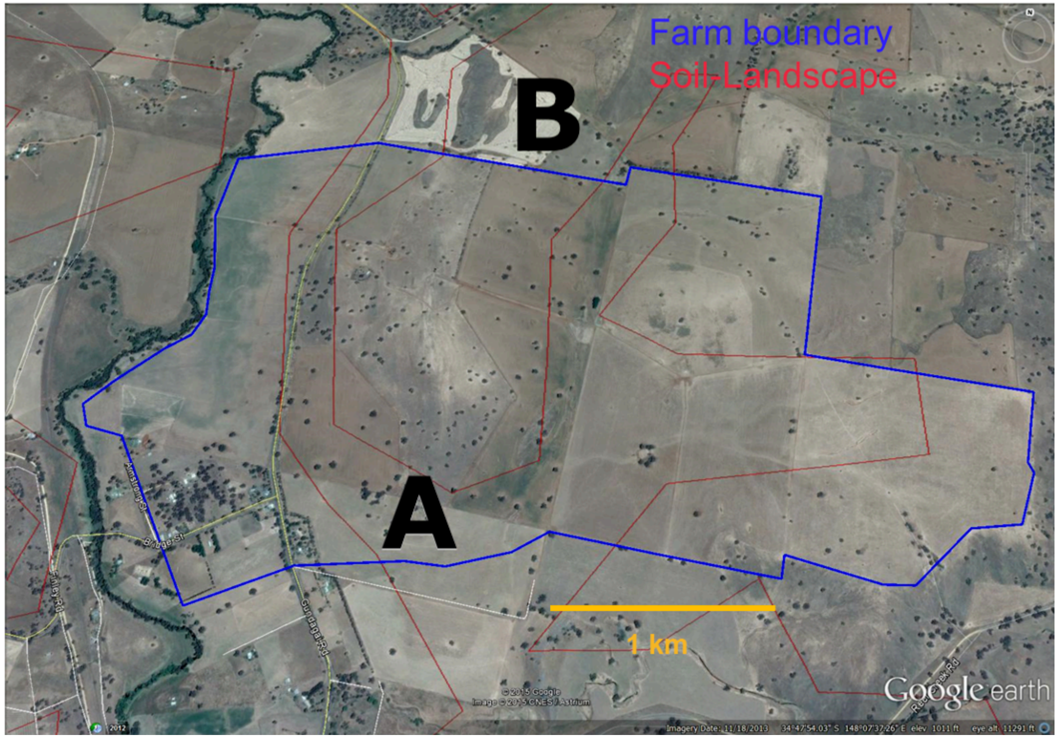
With simple random sampling:

- We expect ~8 samples in grassland, ~2 in wetland
- But by chance, we might get:
    - ➟ 10 grassland, 0 wetland!
    - ➟ Or 6 grassland, 4 wetland

But what if we have more information about the population?

Soil carbon example

Soil carbon



Different land types

- Land type A covers 62% of the area, land type B covers 38%
- Type A has a **higher** chance of being selected with simple random sampling
- **Can we use this information to our advantage?**

~~Simple~~ Stratified random sampling

Stratified random sampling

3 steps

1. **Divide** the population into **homogeneous** subgroups (strata).
2. **Sample** from each stratum using simple random sampling.
3. **Pool** (or **combine**) the estimates from each stratum to get an overall population estimate.

Real-world example

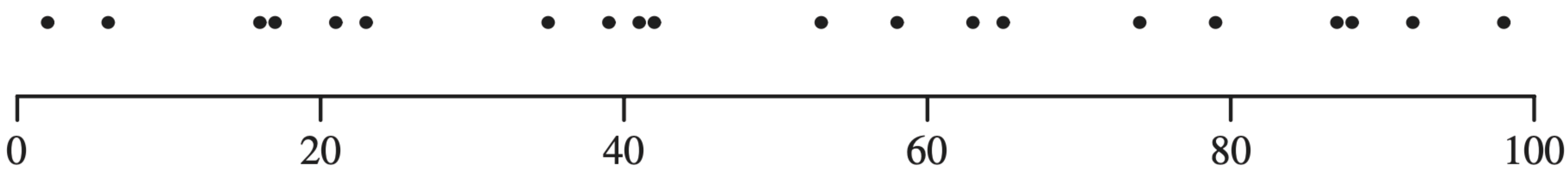
If studying plant biodiversity in a national park:

- Step 1: Divide park into strata (e.g., forest, grassland, wetland)
- Step 2: Take random samples within each habitat type
- Step 3: Combine data to estimate overall biodiversity, giving proper weight to each habitat's area

Strata rules

Strata are…

- **Mutually exclusive and collectively exhaustive** (simple explanation: every sample belongs to exactly one stratum – no overlaps, no leftovers)
- **Homogeneous** - Samples within a stratum should be similar to each other (less variable than the overall population)
- **Each stratum must be sampled** - The goal is to ensure every important group is represented



Stratified random sample of 20 numbers from population of 100 numbers

Good vs. poor stratification choices

Everyday examples

Good strata

- **University students**: Undergrad, Masters, PhD
- **Forest types**: Deciduous, Coniferous, Mixed
- **Income levels**: Low, Medium, High

Poor strata choices

- **Interests**: Sports fans, Music lovers, Foodies (a person can be in multiple groups)
- **Water quality**: Clean, Somewhat polluted (too subjective, not clearly defined)

Advantages

We address:

- **Bias**. Each stratum is sampled, so the sample is representative of the population.
- **Accuracy**. Each stratum is represented by a minimum number of sampling units.
- **Insight**. We can compare strata and make inferences about the population.

Does this make simple random sampling obsolete?

- **No**. *Still* a good technique.
- With large enough samples, the two methods will converge.
- Chance of *not* selecting a unit from a stratum is always there, but reduces as the sample size increases.

Stratified random sampling: estimates

What are we trying to achieve with our calculations?

The statistical journey

Once we have our stratified sample, we need to:

1. **Estimate the population central tendency**: Calculate the pooled mean
2. **Quantify our uncertainty**: Calculate the pooled standard error
3. **Create an inference tool**: Build a confidence interval
4. **Make decisions**: Compare estimates, test hypotheses

All of these steps must account for our stratified design.

The statistical workflow for stratified sampling

Four key steps:

1. **Pooled Mean ($\bar{y}_s$)**: Sum of (stratum weight × stratum mean)
    - Best estimate of the population parameter
2. **Pooled Standard Error**:

$$SE(\bar{y}_s) = \sqrt{\sum w_i^2 \times \frac{s_i^2}{n_i}}$$

    - Accounts for stratum weights and within-stratum variability
3. **t-Critical Value**: Based on $df = n - L$ and α = 0.05
    - Accounts for sample size in uncertainty estimates
4. **Confidence Interval**:

$$\text{Pooled mean} \pm (t - \text{critical} \times SE(\bar{y}_s))$$

    - Range likely containing true population mean

Accounting for strata using “weight”

Weighted estimates

- We need to “weigh” the estimates from each stratum to account for the different stratum sizes and inclusion probabilities.
- Most of the time, we use the stratum size as the weight to calculate **weighted estimates**.
- The *overall* population estimate is the sum of the weighted estimates from each stratum, i.e. we *pool the individual strata information into a single, overall population estimate*.

Example

- A forest contains two types of trees: A and B, with 60% and 40% of the population, respectively.
- We want to estimate the **mean height** of the trees.
- Take **10** height measurements, of which 7 are randomly selected from type A and 3 are randomly selected from type B.
- The **pooled estimate** for the *mean height* of the trees is:

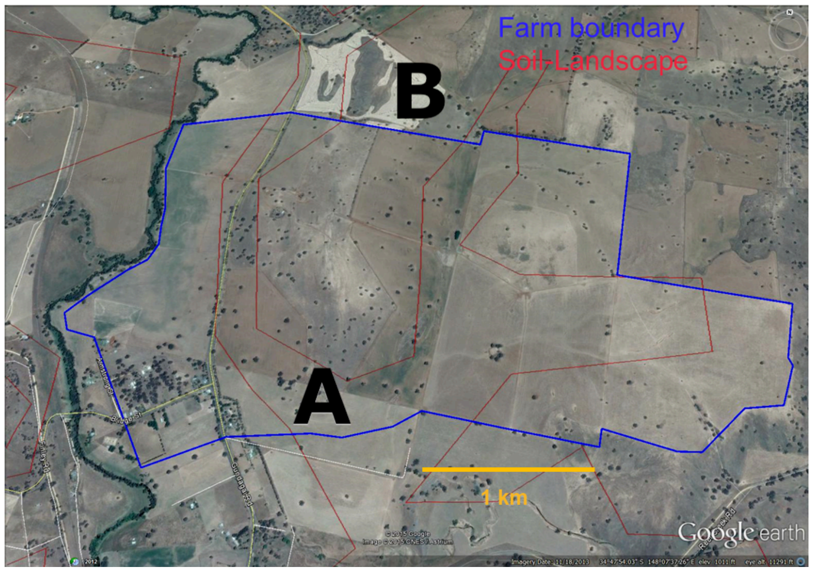
$$0.6 \times \textbf{average height of A} + 0.4 \times \textbf{average height of B}$$

Data story: soil carbon

Soil carbon data

Our case study

Soil carbon content was measured at 7 locations across the area. The amounts were: 48, 56, 90, 78, 86, 71, 42 tonnes per hectare (t/ha).



Setting up the data in R

We know which land type each sample came from:

```r
landA <- c(90, 78, 86, 71)  # stratum A samples (62% of the area)
landB <- c(48, 56, 42)      # stratum B samples (38% of the area)
```

Pooled mean $\bar{y}_s$

> The pooled mean is our best estimate of the overall population mean, taking into account the different stratum sizes.

$$\bar{y}_s = \sum_{i=1}^{L} \bar{y}_i \times w_i$$

In simple terms:

- We calculate the mean for each stratum separately ($\bar{y}_i$)
- We multiply each stratum's mean by its weight (w_i)
- We add these weighted means together to get the overall pooled mean

Calculating pooled mean: soil carbon example

We first define the weights w_i for each stratum based on their area:

▶ Code

Then we calculate the weighted mean:

▶ Code

```
[1] 68.86833
```

This is like saying: "62% of our land has soil carbon like land type A, and 38% has soil carbon like land type B, so our overall estimate takes both into account in these proportions."

Pooled standard error of the mean $SE(\bar{y}_s)$

The formula looks similar to a standard error…

$$SE(\bar{y}_s) = \sqrt{\sum_{i=1}^{L} w_i^2 \times \frac{s_i^2}{n_i}}$$

What's different?

- Instead of a single variance term, we use the sum of weighted variances from each stratum
- The w_i^2 term ensures we account for the relative size of each stratum
- Each stratum contributes its own variance (s_i^2) and sample size (n_i)

t-critical value

Degrees of freedom df

$$df = n - L$$

where n is the total number of samples and L is the number of strata.

- The degrees of freedom tells us how much "free information" we have for making estimates
- For stratified sampling, we lose one degree of freedom for each stratum
- **Example:** If we have 12 samples in 3 strata:
  - The degrees of freedom is $12 - 3 = 9$
  - Think of it this way: 9 samples can be placed anywhere, but we must have at least 1 sample in each of the 3 strata

In R

▶ Code

```
[1] 2.570582
```

95 % Confidence interval for stratified random sampling

The formula

$$95\%\ CI = \bar{y}_s \pm t_{n-L}^{0.025} \times SE(\bar{y}_s)$$

where L is the number of strata, n is the total number of samples, and $\bar{y}_s$ is the weighted mean of the strata.

In simple terms:

- We're creating a range where we're 95% confident the true population mean lies
- We start with our best estimate (the pooled mean $\bar{y}_s$)
- We add and subtract a "margin of error" (which depends on our sample size and variability)
- The margin of error = t-critical value × standard error

Visualising this:

```
Lower bound ← [Pooled mean - Margin of error] ... [Pooled mean + Margin of error] → Upper bound
```

95 % Confidence interval for stratified random sampling

Putting it all together

▶ Code

```
     L95      u95
61.04864 76.68803
```

Comparison

Simple random vs. stratified random sampling

What if we had used stratified random sampling instead of simple random sampling (and collected the same amount of data)?

What differences can you see?

▶ Code

Design	Mean	Var (mean)	L95	U95	df
Simple Random	67.29	50.83	49.85	84.73	6
Stratified Random	68.90	9.30	61.00	76.70	5

Visual comparison of 95% confidence intervals

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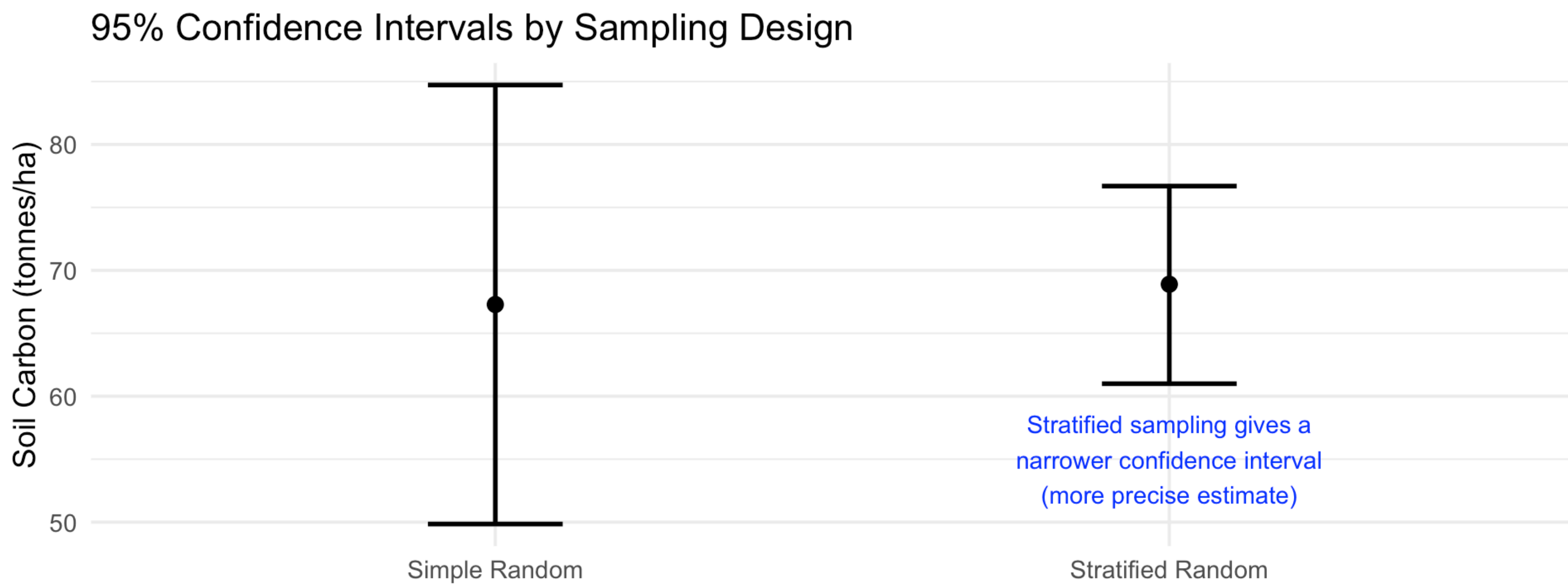

Key insights:

- Both methods give similar estimates of the mean
- Stratified sampling produces a much narrower confidence interval
- The variance of the mean is about 5 times smaller with stratified sampling
- This means stratified sampling is much more precise with the same number of samples

Efficiency

What is sampling efficiency?

- A measure of how much “bang for your buck” you get with different sampling methods
- Calculated as a ratio:

$$\text{Efficiency} = \frac{\text{Variance of SRS}}{\text{Variance of Stratified}}$$

In simple terms:

- Efficiency > 1: Stratified sampling is better (more precise with same sample size)
- Efficiency = 5 means: You’d need 5 times as many samples with simple random sampling to get the same precision as stratified sampling

In R

▶ Code

```
[1] 5.465591
```

How many samples would we have had to collect using simple random sampling to achieve the same precision as our stratified sample?

▶ Code

```
[1] 38
```

So we would need about 38 samples with simple random sampling to get the same precision that we achieved with just 7 samples using stratified sampling!

Tips on implementation

- The most difficult part is to **identify** the strata and **assign** the sampling units to the strata
- Common stratification variables in environmental science:
    - ➟ **Spatial**: elevation bands, soil types, vegetation zones
    - ➟ **Temporal**: seasons, time of day, growth stages
    - ➟ **Management**: treatment types, land-use history
- **Strata sampling size**: allocate samples to strata based on the size of the strata, either proportional to:
    - ➟ the size of the strata (e.g. 60% of area = 60% of samples)
    - ➟ the variance of the strata (more samples where variation is higher)

Monitoring

What if we come back and do another set of soil carbon measurements?

The change in mean $\Delta\bar{y}$

Important considerations

- We want to measure change in soil carbon over time
- Key question: **How do we select sites for the second measurement?**
    1. Return to the **same sites**?
    2. Select completely **new sites**?
- This choice affects our statistical analysis (covariance)

Monitoring estimates

Change in mean $\Delta\bar{y}$

> The difference between the means of the two sets of measurements.

$$\Delta\bar{y} = \bar{y}_2 - \bar{y}_1$$

where $\bar{y}_2$ and $\bar{y}_1$ are the means of the second and first set of measurements, respectively.

Uncertainty in change estimates

Variance *of the change in mean* $Var(\Delta \bar{y})$

This tells us how precise our estimate of the change is. It depends on:

$$Var(\Delta \bar{y}) = Var(\bar{y}_2) + Var(\bar{y}_1) - 2 \times Cov(\bar{y}_2, \bar{y}_1)$$

In simple terms:

- The uncertainty in our change estimate comes from the uncertainties in both measurements
- However, if we sample the same sites twice, they are related to each other (covariance)
- This relationship usually reduces the overall uncertainty in our change estimate

Important: Visiting the same sites twice (paired sampling) usually gives more precise estimates of change than visiting different sites each time!

Covariance and site selection

Quick decision guide

1. **Same sites?** Use paired approach:
    - Sites are the same in both visits
    - Use paired t-test
    - Account for covariance between visits
2. **Different sites?** Use independent approach:
    - New random sites in second visit
    - Use two-sample t-test
    - No covariance between visits

What is covariance?

Covariance measures how two measurements relate to each other:

Example with soil carbon:

- Site 1: First visit = 90 t/ha, Second visit = 95 t/ha
- Site 2: First visit = 48 t/ha, Second visit = 52 t/ha
- Site 3: First visit = 71 t/ha, Second visit = 75 t/ha

What do you notice? Sites with high carbon in the first measurement still have high carbon in the second measurement (positive covariance).

Why this matters: Knowing the first measurement helps us predict the second one, reducing uncertainty in our estimate of change.

Practical takeaway: When measuring change over time, returning to the same sites usually gives more precise results because it removes site-to-site variation.

Calculating the 95% CI for the change in mean

The formula looks similar to before:

$$95\%\ CI = \Delta\bar{y} \pm t_{n-1}^{0.025} \times SE(\Delta\bar{y})$$

In plain language:

- We have our best estimate of the change (the difference between the two means)
- We add and subtract a margin of error to create a range
- We're 95% confident that the true change falls within this range

The standard error of the change $SE(\Delta\bar{y})$

- This tells us how precise our estimate of the change is
- It's complicated to calculate by hand, especially when we visit the same sites twice
- If we visit the same sites twice, we need to account for their relationship (covariance)

Good news! You don't need to calculate this by hand!

- R can do these calculations for you using the `t.test()` function
- For same sites: use `paired = TRUE` option
- For different sites: use `paired = FALSE` option
- We'll practice this in the lab!

Thanks!

Questions?